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On a cusp in the magnetic field–velocity dependence for domain-wall motion in rare-earth orthoferrites

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Abstract. Using the Landau–Lifshitz equations and the equation for acoustic waves, a formula describing the steps in the dependence of domain-wall velocity v on external magnetic field H is obtained. The analysis leads to the conclusion that a 'break' can be observed in the maximum of the $H(v)$ dependence (i.e. a jump in the differential mobility of the domain wall) caused by a change in the contribution of long-wavelength acoustic waves to the dissipation function. An approximation for the $H(v)$ dependence describing such a jump is proposed. The possibility of the existence of a second maximum on the $H(v)$ curve at velocities less than the speed of sound is demonstrated.

Magnetoelastic interaction is known to lead to the appearance of steps in the magnetic field dependence of domain-wall velocity in weak ferromagnets. This has been reviewed both experimentally and theoretically in [1] and discussed in detail in [2]. Domain-wall motion induces an elastic deformation field. This causes an additional magnetic energy dissipation in addition to the ordinary 'viscous' one due to elastic energy dissipation. The conversion of magnetic energy into sound energy occurs at a maximum rate when the domain-wall velocity is equal to the speed of sound. Therefore, as the former approaches the latter, the drag force acting on the domain wall increases. As a result, 'steps' (regions of small slope) appear in the magnetic field dependence of the domain-wall velocity, the locations of the steps coinciding with sound speeds [1, 2].

In the present paper macroscopic equations of motion of the simplest one-dimensional model taking this effect into account are given. The formula for resonant energy loss is obtained starting from the energy balance equation and is similar to formulae of [2]. Analysing this formula, it is concluded that the differential mobility of the domain walls changes abruptly when the domain-wall velocity equals the sound velocity. This jump in mobility is caused by an increasing contribution of long-wavelength acoustic waves to the energy dissipation when the domain-wall velocity approaches the speed of sound. In order to describe the jump in domain-wall mobility accurately, we write a drag force approximation differing substantially from the Lorentzian proposed in [2].

Consider a two-sublattice weak ferromagnet, with the magnetizations of the sublattices denoted as M_1 and M_2, with $|M_1| = |M_2| = M_0$. The vectors of ferromagnetism and antiferromagnetism are defined conventionally as

$$m = (M_1 + M_2)/2M_0 \qquad l = (M_1 - M_2)/2M_0$$

$$(\boldsymbol{m})^2 + (\boldsymbol{l})^2 = 1 \qquad (\boldsymbol{m} \cdot \boldsymbol{l}) = 0. \tag{1}$$

We take as the energy density for a weak ferromagnet (see for example [3])†:

$$w = M_0^2\{\tfrac{1}{2}\delta(\boldsymbol{m})^2 + \tfrac{1}{2}\alpha(\nabla \boldsymbol{l})^2 + \tfrac{1}{2}Bl_y^2 - \tfrac{1}{2}\beta l_x^2 + [\boldsymbol{d}\cdot(\boldsymbol{l}\times\boldsymbol{m})] - 2(\boldsymbol{m}\cdot\boldsymbol{h}) + f_{ij,kn}(l_i l_j - l_i^{(0)} l_j^{(0)})u_{kn}\} + \tfrac{1}{2}\lambda_{ij,kn}u_{ij}u_{kn} \tag{2}$$

where u_{ij} is a deformation tensor, $\lambda_{ij,kn}$ and $f_{ij,kn}$ are tensors of elastic and magnetoelastic constants respectively, $\boldsymbol{h} = h\boldsymbol{e}_z$ is the normalized magnetic field, $\boldsymbol{d} = d\boldsymbol{e}_y$ with d being the Dzyaloshinskii constant, $B > 0$ and $\beta > 0$ are magnetic anisotropy constants, and δ and α are exchange constants. Let us also assume the dissipation function:

$$F = \tfrac{1}{2}\int\left(\frac{A}{\gamma M_0}[(\boldsymbol{M}_1\times\boldsymbol{H}_1)^2 + (\boldsymbol{M}_2\times\boldsymbol{H}_2)^2] + \rho\Gamma_{ij,kn}\dot{u}_{ij}\dot{u}_{kn}\right)\mathrm{d}V \tag{3}$$

with $\boldsymbol{H}_i = \gamma\,\delta W/\delta\boldsymbol{M}_i$ ($i = 1, 2$) being effective fields, $W = \int w\,\mathrm{d}V$ the internal energy, A the damping constant of spin waves, γ the gyromagnetic ratio, $\Gamma_{ij,kn}$ the tensor of damping constants of sound, and ρ the mass density of the material. The orders of magnitude of material constants are assumed to be close to those in rare-earth orthoferrites: $M_0 \sim 10^3$ G, $\delta \sim 10^3$–10^4, $\alpha \sim \delta a^2$ with $a \sim 10^{-7}$ cm being the crystal lattice parameter, $d \sim 1$–10, $B \geqslant \beta$, $\beta \sim 0.1$–1, $f \sim 10$, the sound velocity $s_0 \sim 10^5$ cm s^{-1}, $A \sim 10^{-5}$–10^{-4}, the natural time unit $2/(\gamma M_0) \sim 10^{-10}$ s, $\rho \sim 10$ g cm^{-3} and $\Gamma \sim 1$ cm^2 s^{-1}. The Landau–Lifshitz equations including relaxation terms $\boldsymbol{R}_i = -\gamma\,\delta F/\delta\boldsymbol{H}_i$ [4, 5] have the form

$$\partial\boldsymbol{M}_i/\partial t = (\boldsymbol{M}_i\times\boldsymbol{H}_i) - \gamma\,\delta F/\delta\boldsymbol{H}_i \qquad i = 1,2. \tag{4}$$

Following [6–10] we can use the relation

$$\boldsymbol{m} = (1/\delta)\{2[\boldsymbol{h} - \boldsymbol{l}(\boldsymbol{l}\cdot\boldsymbol{h})] + (\boldsymbol{l}\times\boldsymbol{d}) - (\boldsymbol{l}\times\dot{\boldsymbol{l}})\} \tag{5}$$

to eliminate $\boldsymbol{m}$, and reduce (4) (in the long-wavelength limit) to the single motion equation of unit vector $\boldsymbol{l}$ ($|\boldsymbol{m}| \ll |\boldsymbol{l}| \simeq 1$). This equation can be written in the following Lagrangian form [8–10]:

$$\left[\boldsymbol{l}\times\left(\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\delta L}{\delta\dot{\boldsymbol{l}}}\right) - \frac{\delta L}{\delta\boldsymbol{l}} + \frac{1}{M_0^2}\frac{\delta F}{\delta\dot{\boldsymbol{l}}}\right)\right] = 0. \tag{6}$$

The Lagrangian L contains magnetic, elastic and magnetoelastic terms:

$$L = L_{\mathrm{M}} + L_{\mathrm{E}} + L_{\mathrm{ME}} \tag{7}$$

with

$$L_{\mathrm{M}} = \int\left(\frac{1}{2\delta}(\dot{\boldsymbol{l}})^2 - \frac{\alpha}{2}(\nabla\boldsymbol{l})^2 - \tilde{w}_a(\boldsymbol{l}) + \frac{2}{\delta}[\boldsymbol{h}\cdot(\dot{\boldsymbol{l}}\times\boldsymbol{l})] - \frac{2}{\delta}[\boldsymbol{l}\cdot(\boldsymbol{h}\times\boldsymbol{d})]\right)\mathrm{d}V$$

† In this expression only the anisotropic exchange term is taken into account; the small spin–orbital energy $d'(m_x l_z + m_z l_x)$ is neglected.

where

$$\bar{w}_a(\boldsymbol{l}) = \frac{B}{2}l_y^2 - \frac{\beta}{2}l_x^2 + \frac{1}{2\delta}(\boldsymbol{l}\cdot\boldsymbol{d})^2 + \frac{2}{\delta}(\boldsymbol{l}\cdot\boldsymbol{h})^2$$

$$L_{\rm E} = \int \left[\frac{\rho}{2}\left(\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t}\right)^2 - \tfrac{1}{2}\lambda_{ij,kn}u_{ij}u_{kn}\right]\mathrm{d}V$$

$$L_{\rm ME} = -\int f_{ij,kn}(l_il_j - l_i^{(0)}l_j^{(0)})u_{kn}\,\mathrm{d}V.$$

The acoustic wave equation is written by means of Lagrangian (7) as follows:

$$(\mathrm{d}/\mathrm{d}t)(\delta L/\delta\dot{\boldsymbol{u}}) - \delta L/\delta\boldsymbol{u} + \delta F/\delta\dot{\boldsymbol{u}} = 0. \tag{8}$$

Equation (8) is an inhomogeneous linear equation for the components of the deformation vector $\boldsymbol{u}$. Its solution is

$$u_i = \int G_{ij}(\boldsymbol{r}-\boldsymbol{r}', t-t')f_{kn,jp}\frac{\partial}{\partial x'_p}(l_k(\boldsymbol{r}',t')l_n(\boldsymbol{r}',t'))\,\mathrm{d}V'\,\mathrm{d}t' \tag{9}$$

where $G_{ij}(\boldsymbol{r}-\boldsymbol{r}', t-t')$ is the Green function tensor of the equation

$$(\mathrm{d}/\mathrm{d}t)(\partial L_{\rm E}/\delta\dot{\boldsymbol{u}}) - \delta L_{\rm E}/\delta\boldsymbol{u} + \delta F/\delta\dot{\boldsymbol{u}} = 0. \tag{10}$$

Substituting expression (9) into equation (6) we obtain an equation for the distribution of magnetization in a domain wall.

We apply this general theory further to a particular case of a one-dimensional domain wall in a weak ferromagnet (7) travelling along the y axis. Let also $B \gg \beta$. In this case

$$l_x = \cos\theta \qquad l_y = 0 \qquad l_z = \sin\theta$$

and equations (6) and (8) reduce to

$$(\alpha - v^2/\delta)\mathrm{d}^2\theta/\mathrm{d}\xi^2 + Av\,\mathrm{d}\theta/\mathrm{d}\xi - (2hd/\delta)\sin\theta - (\beta/2)\sin(2\theta) = f(\mathrm{d}u/\mathrm{d}\xi)\sin(2\theta) \tag{11}$$

$$\Gamma v\,\mathrm{d}^3u/\mathrm{d}\xi^3 + (v^2 - s_0^2)\,\mathrm{d}^2u/\mathrm{d}\xi^2 = (fM_0^2/\rho)(\mathrm{d}/\mathrm{d}\xi)(\sin^2\theta) \tag{12}$$

where $\theta = \theta(\xi)$, $u = u(\xi)$, $\xi = y - vt$, $u = u_y$, $f = f_{zz,yy} - f_{xx,yy}$, $\Gamma = \Gamma_{yy,yy}$, and $s_0 = \surd(\lambda_{yy,yy}/\rho)$ is the velocity of longitudinal sound waves. The following expression for $\mathrm{d}u/\mathrm{d}\xi$ is derived from equation (12) by means of the Green function:

$$\frac{\mathrm{d}u}{\mathrm{d}\xi} = -\frac{fM_0^2}{2\pi\rho}\int_{-\infty}^{+\infty}\frac{\exp(\mathrm{i}k\xi)}{(s_0^2 - v^2) - \mathrm{i}\Gamma kv}\mathrm{d}k\int_{-\infty}^{+\infty}\sin^2\theta(\eta)\exp(-\mathrm{i}k\eta)\,\mathrm{d}\eta. \tag{13}$$

An equation for the function $\theta(\xi)$ is obtained by substituting (13) into (11):

$$\begin{aligned}(1 - v^2/\alpha\delta)\,\mathrm{d}^2\theta/\mathrm{d}\xi^2 + (Av/\alpha)\,\mathrm{d}\theta/\mathrm{d}\xi - (2hd/\alpha\delta)\sin\theta - (\beta/2\alpha)\sin(2\theta)\\ = -\frac{f^2M_0^2}{2\pi\alpha\rho}\sin(2\theta)\int_{-\infty}^{+\infty}\frac{\exp(\mathrm{i}k\xi)}{(s_0^2 - v^2) - \mathrm{i}\Gamma kv}\,\mathrm{d}k\\ \times\int_{-\infty}^{+\infty}\sin^2\theta(\eta)\exp(-\mathrm{i}k\eta)\,\mathrm{d}\eta.\end{aligned} \tag{14}$$

Depending on the value of the parameter Γ, either the inequality $(f^2M_0^2/\beta\rho s_0)\sqrt{(\alpha/\beta)} \gtrsim \Gamma$ or the reverse one $(f^2M_0^2/\beta\rho s_0)\sqrt{(\alpha/\beta)} \ll \Gamma$ holds. In the first case, the order of magnitude of the right-hand side of equation (14) is comparable to that of the last term on the left (see appendix). As a consequence, the question arises of the existence of a solution in the form of a domain wall at $v \simeq s_0$, where the deformation $\mathrm{d}u/\mathrm{d}\xi$ reaches its maximum. Therefore a gap can appear in the velocity spectrum of the domain wall. This fact is evident in the case $\Gamma = 0$ when the integral in (13) is transformed into $\sin^2\theta$ up to a constant factor and integro-differential equation (14) is reduced to a differential one†. Values of material constants chosen in the present paper satisfy $(f^2M_0^2/\beta\rho s_0)\sqrt{(\alpha/\beta)} \ll \Gamma$. In this case the gap in the velocity spectrum does not appear since, for any value of the variable θ, the last term on the left of equation (14) is much larger than the right-hand term.

Substituting (13) into the relation

$$\mathrm{d}W/\mathrm{d}t = -2F$$

one obtains the well known energy balance equation

$$h = \frac{Av\delta}{4d}\int_{-\infty}^{+\infty}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right)^2 \mathrm{d}\xi + \frac{f^2M_0^2\delta}{8\pi\rho\Gamma v d}\int_{-\infty}^{+\infty}\frac{k^2}{[(s_0^2-v^2)/(\Gamma v)]^2+k^2}\,\mathrm{d}k \times\left|\int_{-\infty}^{+\infty}\sin^2\theta(\eta)\exp(\mathrm{i}k\eta)\,\mathrm{d}\eta\right|^2. \tag{15}$$

The integral in the second term of (15) has the form:

$$I(k_0) = \int_{-\infty}^{+\infty}\frac{k^2 f(k)}{k_0^2+k^2}\,\mathrm{d}k \tag{16}$$

where

$$k_0 = \frac{s_0^2 - v^2}{\Gamma v} \qquad f(k) = \left|\int_{-\infty}^{+\infty}\sin^2\theta(\eta)\exp(\mathrm{i}k\eta)\,\mathrm{d}\eta\right|^2.$$

In the limit of large $k_0\Delta$, $\Delta = \sqrt{[(\alpha\delta - v^2)/\beta\delta]}$ being the thickness of the domain wall, integral (16) can be expanded in an asymptotic series:

$$I(k_0) = \int_{-\infty}^{+\infty} f(k)\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{k^2}{k_0^2}\right)^n \mathrm{d}k = \frac{1}{k_0^2}\int_{-\infty}^{+\infty}k^2 f(k)\,\mathrm{d}k + \mathrm{O}\left(\frac{1}{(k_0\Delta)^4}\right).$$

In the opposite case, $k_0\Delta$ being small, this integral is expanded as follows:

$$\begin{aligned}
I(k_0) &= \int_{-\infty}^{+\infty}\frac{k^2 f(k)}{k_0^2+k^2}\,\mathrm{d}k = \int_{-\infty}^{+\infty} f(k)\,\mathrm{d}k - k_0^2\int_{-\infty}^{+\infty}\frac{f(k)}{k_0^2+k^2}\,\mathrm{d}k \\
&= \int_{-\infty}^{+\infty} f(k)\,\mathrm{d}k - k_0^2 f(0)\int_{-\infty}^{+\infty}\frac{\mathrm{d}k}{k_0^2+k^2} - k_0^2 \\
&\quad\times\int_{-\infty}^{+\infty}\frac{k^2}{k_0^2+k^2}\left(\frac{f(k)-f(0)}{k^2}\right)\mathrm{d}k \\
&= \int_{-\infty}^{+\infty} f(k)\,\mathrm{d}k - \pi f(0)|k_0| - k_0^2\int_{-\infty}^{+\infty}\frac{f(k)-f(0)}{k^2}\,\mathrm{d}k + \mathrm{O}((k_0\Delta)^3)
\end{aligned}$$

† Uchiyama *et al* [11] have investigated this differential equation previously and proved the existence of the gap in the velocity spectrum of the domain wall. Zvezdin and Popkov [12] have also studied the case $\Gamma \neq 0$.

where we take into account that

$$f(k)=\left|\int_{-\infty}^{+\infty}\sin^2\theta(\eta)\exp(\mathrm{i}k\eta)\,\mathrm{d}\eta\right|^2$$

is even and hence may be expanded in an asymptotic power series in k^2. The series coefficients at even powers of k_0 are integrals regularized in the sense of generalized function theory. At $f(0)\neq 0$† the maximum point of the dependence $h(v)$ is its cusp (a discontinuity of its first derivative), with jump in derivative described by the term $-\pi f(0)|k_0|$.

In order to elucidate the cause of this singularity, let us remark that according to formula (13)

$$\frac{\mathrm{d}u}{\mathrm{d}\xi}=\int_{-\infty}^{+\infty}A(k)\exp(\mathrm{i}k\xi)\,\mathrm{d}k$$

where Fourier transform $A(k)$ is expressed as

$$A(k)=U(k)/(k+\mathrm{i}k_0)$$

the function $U(k)$ depending on the structure of the domain wall. It is at once apparent that, in the limit of small k, $A(k)\simeq U(k)/k$ in the case $k_0=0$ and otherwise (when $k_0\neq 0$) $A(k)\simeq U(k)/\mathrm{i}k_0$. It is also evident that at $|k|\gg|k_0|$ the Fourier transform $A(k)$ depends weakly on the variable k_0. The dissipation function of elastic waves can be written to within a factor independent of this variable as follows:

$$F\sim\int_{-\infty}^{+\infty}|A(k)|^2k^2\,\mathrm{d}k.$$

For small k the spectral density of the dissipation function under the condition $k_0=0$ is $|A(k)|^2k^2\simeq|U(0)|^2$. On the other hand, in the case $k_0\neq 0$ one can put $|A(k)|^2k^2\simeq|U(0)|^2k^2/k_0^2$ over the range $-|k_0|<k<|k_0|$. Thus the dissipation function for small but non-zero values of the parameter k_0 is different from that for $v=s_0$ (i.e. $k_0=0$) by

$$|U(0)|^2\int_{-|k_0|}^{|k_0|}\left(\frac{k^2}{k_0^2}-1\right)\mathrm{d}k=-\tfrac{4}{3}|U(0)|^2|k_0|.$$

In other words, the jump in the domain-wall mobility at $v=s_0$ is caused by a change in the contribution of long-wavelength acoustic waves to the dissipation function.

† Solution (13) of equation (12) meets the condition $f(0)\neq 0$ because of the appropriate structure of equations (11) and (12). However, in the general case

$$f(k)=\left|\int_{-\infty}^{+\infty}f_{ij,kn}l_i(\xi)l_j(\xi)\exp(\mathrm{i}k\xi)\,\mathrm{d}\xi\right|^2$$

and it is clear that this expression depends on the symmetry of the magnetic material and the direction of domain-wall movement. Therefore, the case $f(0)=0$ is actually possible.

The order of magnitude of the above-mentioned characteristic wavelength can be evaluated as follows. Under the conditions $|s_0 - v| \sim 10\ \mathrm{m\ s^{-1}} = 10^3\ \mathrm{cm\ s^{-1}}$ and $\Gamma \sim 1\ \mathrm{cm^2\ s^{-1}}$, the latter corresponding to rare-earth orthoferrites at room temperature, the dissipation function decreases over the spectral range $|k| < 2|s_0 - v|/\Gamma \sim 2\times 10^3\ \mathrm{cm^{-1}}$, the respective wavelength being more than 30 μm.

In [2] the Lorentzian (formula (22) of [2]) has been substituted for the delta function in the expression for the drag force acting on the domain wall (formula (13) of [2]). This transformation corresponds to formula (15) of our paper. The authors of [2] have assumed that integral (16) of our paper can be approximated by a Lorentzian. It is clear, however, that no Lorentz approximation allows the description of the above jump in differential mobility. The formulae given in [2] must be refined in this respect.

This refined approximation can be obtained by constructing the succession of Padé approximations for integral (16), i.e. rational approximations, with values of their coefficients being derived from the condition that the first few coefficients of their Taylor series are equal to those of the approximated function (16)†.

Let us assume initially that $f(0) = 0$. In this case, $k_0 = 0$ is a parabolic maximum point of integral (16). The respective Padé approximation must be maximized at the same point with just the same maximum value. In the limit $k_0 \to \infty$ coefficients at k_0^{-2} and k_0^{-3} in the asymptotic series of the Padé approximation have to be equal to those of (16). The Lorentz approximation proposed in [2] is evidently the simplest one.

$$h \simeq (Av\delta/4d)P + (f^2M_0^2\delta/8\pi\rho\Gamma vd)QR/(k_0^2Q + R) \tag{17}$$

where

$$P = \int_{-\infty}^{+\infty}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right)^2\mathrm{d}\xi \qquad Q = 2\pi\int_{-\infty}^{+\infty}\left[f_{ij,kn}l_i(\xi)l_j(\xi)\right]^2\mathrm{d}\xi$$

$$R = 2\pi\int_{-\infty}^{+\infty}\left(\frac{\mathrm{d}}{\mathrm{d}\xi}[f_{ij,kn}l_i(\xi)l_j(\xi)]\right)^2\mathrm{d}\xi$$

the integration variable k in the integrals Q and R being substituted by ξ with equality

$$\int_{-\infty}^{+\infty}\left|\int_{-\infty}^{+\infty}\Phi(x)\exp(ikx)\,\mathrm{d}x\right|^2\mathrm{d}k = 2\pi\int_{-\infty}^{+\infty}|\Phi(x)|^2\,\mathrm{d}x.$$

Now let $f(0) \neq 0$. As mentioned above, in this case integral (16) is a series in integer powers of variable $|k_0|$ but not k_0^2. The three-point Padé approximation of integral (16) is constructed as a rational function of variable $|k_0|$, with its maximum value, the Taylor coefficient at $|k_0|$, and the latter at k_0^2 being accurate. One assumes also in the limit $k_0 \to \infty$ the asymptotic series coefficient at k_0^{-2} to be accurate and that at $|k_0|^{-3}$ to vanish. These conditions can be met with the following function:

$$h \simeq \frac{Av\delta}{4d}P + \frac{f^2M_0^2\delta}{8\pi\rho\Gamma vd}\,\frac{Q + \Lambda R|k_0|}{1 + [(S + \Lambda R)/Q]|k_0| + (Q/R)k_0^2 + \Lambda|k_0|^3} \tag{18}$$

† Constructing the succession of Padé approximations can be considered as a procedure to calculate integral (16). The first few Padé approximations turn out to give the value of this integral with a fair accuracy.

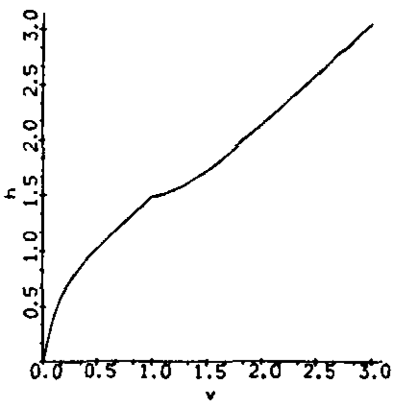

Figure 1. The dependence $H_0(v/c_0)$ of non-hysteresis type; $D = 0.5$, $F_0 = 0.15$.

where

$$S = \pi \left(\int_{-\infty}^{+\infty} \sin^2 \theta(\xi)\, \mathrm{d}\xi \right)^2 \qquad \Lambda = \frac{QJ}{RS} + \frac{Q^3}{R^2 S} - \frac{S}{R}$$

$$J = \int_{-\infty}^{+\infty} \frac{f(0) - f(k)}{k^2}\, \mathrm{d}k.$$

Assuming parameters A, f^2 and h in equation (14) to be small, we can formulate a perturbation theory in A and f^2. For the kink-type solution describing a domain wall, the perturbation theory has to be formulated for variable $V = \mathrm{d}\theta/\mathrm{d}\xi = V(\theta)$, where $0 \leqslant \theta \leqslant \pi$. In such a case V and h are expressed by perturbation theory series as

$$V(\theta) = V_0(\theta) + V_1(\theta) + V_2(\theta) + \ldots \qquad h = h_1 + h_2 + \ldots$$

the zero-order approximation $V_0(\theta)$ corresponding to a kink of the sine–Gordon equation†. The condition for the absence of secular terms in the expression for $V_1(\theta)$ can be written in the form of the energy balance equation (15), h and $\theta(\xi)$ being substituted respectively by h_1 and zero-order approximation $\theta_0(\xi)$ written as

$$\theta(\xi) \simeq \theta_0(\xi) = \pi - \cos^{-1} \tanh \left[\xi \sqrt{\left(\frac{\beta\delta}{\alpha\delta - v^2} \right)} \right].$$

Thus we obtain

$$h \simeq h_1 = \frac{A\sqrt{\beta}\,\delta^{3/2} v}{2d\sqrt{(\alpha\delta - v^2)}} + \frac{\pi f^2 M_0^2 \Gamma v (\alpha\delta - v^2)^2}{8\rho\beta^2\delta d} \times \int_{-\infty}^{+\infty} \frac{k^4}{(s_0^2 - v^2)^2 + \Gamma^2 k^2 v^2} \sinh^{-2} \left[\frac{\pi k}{2} \sqrt{\left(\frac{\alpha\delta - v^2}{\beta\delta} \right)} \right] \mathrm{d}k. \tag{19}$$

The approximation (18) for dependence $h(v)$ can be written accordingly as follows:

$$h \simeq h_1 \simeq \frac{A\sqrt{\beta}\,\delta^{3/2} v}{2d\sqrt{(\alpha\delta - v^2)}} + \frac{f^2 M_0^2 \delta}{3\rho\Gamma v d} \Delta \frac{1 + \frac{1}{3}\Delta |k_0|}{1 + \frac{11}{6}\Delta |k_0| + \frac{5}{4}\Delta^2 k_0^2 + \frac{5}{12}\Delta^3 |k_0|^3}. \tag{20}$$

† Formulae of this perturbation theory derived for another problem have been written in the appendix of [13].

The dependence $h_1(v)$ in normalized units for a number of values of the model parameters is shown in figures 1–3. Padé approximation (20) gives coincidence with (19) within 2% accuracy for all three dependences; therefore, only one curve is shown. The normalized units are derived from such a form of formula (19):

$$H_1 = h_1 \frac{2d\sqrt{(\alpha\delta - v^2)}}{A\beta^{1/2}\delta^{3/2}s_0} = \frac{v}{s_0} + F_0 \frac{s_0}{v} \int_{-\infty}^{+\infty} \frac{z^4\,\mathrm{d}z}{\{[D(v/s_0 - s_0/v)]^2 + z^2\}\sinh^2 z}$$

where

$$F_0 = \frac{2f^2 M_0^2(\alpha\delta - v^2)}{\pi^2\rho\Gamma s_0^2 A\beta\delta} \qquad D = \frac{\pi s_0}{2\Gamma}\sqrt{\left(\frac{\alpha\delta - v^2}{\beta\delta}\right)}.$$

The calculations were carried out with the condition $v^2 \ll \alpha\delta$. It is seen that, depending on the relationship between model parameters, there can exist a hysteresis of the dependence $v(h)$ (this fact was discussed in [2]). This hysteresis means in practice that there is a non-stationary motion of the domain wall within the region of the hysteresis loop [1].

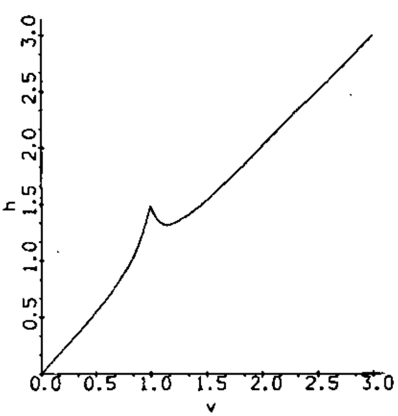

Figure 2. The dependence $H_0(v/c_0)$ of hysteresis type; $D = 4$, $F_0 = 0.15$.

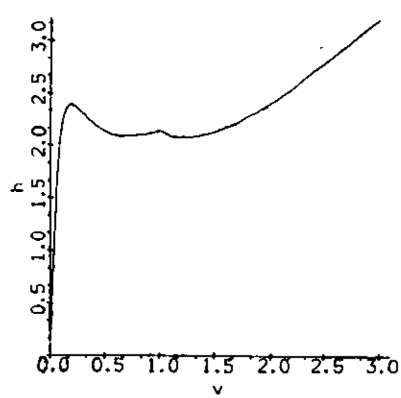

Figure 3. The dependence $H_0(v/c_0)$ with two maxima; $D = 0.25$, $F_0 = 0.35$.

It is also of interest that, depending upon the relationship of F_0 and D, the second maximum at $v < s_0$ can appear due to the factor $1/v$ in the second term of (15). As seen from expression (13), besides a decrease of deformation with increasing domain-wall velocity, there is also a resonant increase of the deformation as $k_0 \to 0$, the former and the latter being described respectively by factor $1/v$ and denominator $1/(k + \mathrm{i}k_0)$. Within some region of the parameters F_0 and D, the competition between these effects over the range $0 < v < s_0$ causes the second maximum (at $v < s_0$) to appear. Providing there is no jump in mobility of the domain wall, i.e. in the case $U(0) = 0$, the same factor $1/v$ causes the resonant velocity to become smaller than s_0; the effect is similar to a shift of normal frequency of a harmonic oscillator with damping. The appearance of the second maximum in the $h(v)$ dependence leads to the second instability region at $v < s_0$ (or even $v \ll s_0$). Such an instability was observed in experiments [14], and we think that the mechanism presented above can serve as one possible explanation (besides the mechanisms proposed earlier, see e.g. [15]).

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Appendix

It is known [12] that a gap can appear in the velocity spectrum of the domain wall only for sufficiently small dissipation of acoustic waves. In our consideration this corresponds to the case when the right-hand side of equation (14) is comparable to (or larger than) the last term on the left. Since both terms under consideration contain common factor $\sin(2\theta)$, we shall compare only the coefficient $\beta/(2\alpha)$ with the following integral:

$$\frac{f^2 M_0^2}{2\pi\alpha\rho}\int_{-\infty}^{+\infty}\frac{\exp(\mathrm{i}k\xi)\,\mathrm{d}k}{(s_0^2-v^2)-\mathrm{i}\Gamma kv}\int_{-\infty}^{+\infty}\sin^2\theta(\eta)\exp(-\mathrm{i}k\eta)\,\mathrm{d}\eta. \tag{A1}$$

The value of the latter can be estimated using an asymptotic series similar to that for integral (16). In the case $|k_0\Delta|\gg 1$ we obtain

$$\int_{-\infty}^{+\infty}\frac{\exp(\mathrm{i}k\xi)\,\mathrm{d}k}{(s_0^2-v^2)-\mathrm{i}\Gamma kv}\int_{-\infty}^{+\infty}\sin^2\theta(\eta)\exp(-\mathrm{i}k\eta)\,\mathrm{d}\eta$$
$$=2\pi\frac{\sin^2\theta(\eta)}{s_0^2-v^2}\left[1+\mathrm{O}\left(\frac{1}{k_0\Delta}\right)\right].$$

Substituting this equality into (A1) we obtain the condition for the absence of a gap in the velocity spectrum in the case $|k_0\Delta|\gg 1$ as

$$\left|\frac{f^2M_0^2\sin^2\theta(\xi)}{\alpha\rho(s_0^2-v^2)}\right|\ll\frac{\beta}{2\alpha}. \tag{A2}$$

Checking (A2) with the condition for its use:

$$|k_0\Delta|=\left|\frac{s_0^2-v^2}{\Gamma v}\right|\sqrt{\left(\frac{\alpha\delta-v^2}{\beta\delta}\right)}\simeq\left|\frac{s_0^2-v^2}{\Gamma s_0}\right|\sqrt{\left(\frac{\alpha}{\beta}\right)}\gg 1$$

and considering inequality $|\sin^2\theta(\xi)|\leqslant 1$ one concludes that fulfilment of the inequality

$$\Gamma\gg(f^2M_0^2/\beta\rho s_0)\surd(\alpha/\beta) \tag{A3}$$

results in fulfilment of condition (A2) at $|k_0\Delta|\gg 1$.

On the other hand, consideration of the series expansion of integral (A1) under condition $|k_0\Delta| \ll 1$ leads to the following order-of-magnitude estimate:

$$\int_{-\infty}^{+\infty} \frac{\exp(\mathrm{i}k\xi)\,\mathrm{d}k}{(s_0^2 - v^2) - \mathrm{i}\Gamma k v} \int_{-\infty}^{+\infty} \sin^2\theta(\eta) \exp(-\mathrm{i}k\eta)\,\mathrm{d}\eta$$
$$= \frac{2\pi}{\Gamma v}\bigg([\Theta_{\mathrm{H}}(-k_0\xi)\exp(-|k_0\xi|)\mathrm{sgn}(k_0) + \Theta_{\mathrm{H}}(\xi)] \int_{-\infty}^{+\infty} \sin^2\theta(\eta)\,\mathrm{d}\eta$$
$$- \int_{-\infty}^{\xi} \sin^2\theta(\eta)\,\mathrm{d}\eta\bigg)[1 + \mathrm{O}(k_0\Delta)] \sim \frac{2\pi}{\Gamma s_0} \int_{-\infty}^{+\infty} \sin^2\theta(\eta)\,\mathrm{d}\eta \sim \frac{2\pi\Delta}{\Gamma s_0}$$

where $\Theta_{\mathrm{H}}(\xi)$ is the Heaviside function. Substituting this estimate into (A1) results in the following condition for the absence of a gap in the velocity spectrum in the case $|k_0\Delta| \ll 1$:

$$f^2 M_0^2 \Delta / \alpha\rho\Gamma s_0 \simeq (f^2 M_0^2/\alpha\rho\Gamma s_0)\surd(\alpha/\beta) \ll \beta/2\alpha$$

which is identical with inequality (A3). Thus inequality (A3) is a sufficient condition for the absence of a gap in the velocity spectrum in both cases $|k_0\Delta| \gg 1$ and $|k_0\Delta| \ll 1$. Therefore no gap in the velocity spectrum of the domain wall appears if (A3) holds.

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